Systematization of Equilibrium and Unstable Bloch Domain Walls in

**Plates of Cubic Ferromagnetic Crystals** 

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The energy and structure of planar Bloch domain walls with opposite directions (right- and left-

handed) and different paths of magnetization vector rotation have been systemized. The theory covers

arbitrary oriented plates of cubic m3m symmetry magnetically ordered medium with negative anisotropy.

The general approach is created and applied to most plates typically used in the experiments.

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1. Introduction

The first important path of the process of the inhomogeneous magnetization reversal (IMR) in the

real crystals is the domain nucleation from the local magnetization inhomogeneity (0° domain wall).

These processes have been investigated in details [1]. The appearing of the new phase domain nucleus in

the volume of the domain walls (DWs) is another important mechanism of IMR processes [2]. Then, the

growth of the DW width  $\delta$  [3] is the important mechanism of the magnetic reversal. It was determined

that specific Bloch DW orientations correspond to the "infinity" width of the DW [3]. Consequently,

general investigation of the DW structure as well as all possibilities of its changes (including nucleation

and DW decay in the limit case) is the important problem for fundamental research and applications of

the magnetically ordered media.

There are many works on the investigations of the DW structure [3-11]. Planar Bloch DWs [4]

have been investigated in the unrestricted crystals [3-6] as well as in the restricted bulk (i.e., in cases

when the crystal sample width is sufficiently larger [5] than DW one) crystals [6-11] with the negative

cubic magnetocrystalline anisotropy. The spatial restriction of the volume of magnetically ordered medium leads to the reflection of the DW plain from its orientation which were realized in the unrestricted crystal and to growth of their energy density in general case [6,9,11]. In fact, only very specific cases of DWs were investigated in details for cases of bulk plates. They are mainly DWs with lowest energies in high-symmetrical oriented plates: (110) [6,9-11], (100) [12-14] and (111) [15,16]. The sequel analysis of domain structure of these samples requires investigation of all possible DWs in their volume. The same investigation are also required for sample with remained orientations (examples are widely used (112) [17] and (210)- [18] plates and films) as well as disoriented samples [6].

In our opinion, general and detailed systematization of the DWs in the restricted samples (plates) was not provided yet. General symmetry classification of the magnetic DWs was done [19,20]. To complete these investigations it is necessary to solve variation problem to found out numerically the thermodynamically equilibrium state of the DW structure for each case of the DW symmetry.

The aim of this work is building of the systematization of energy and structure of planar Bloch DWs with opposite directions (right- and left-handed) and different paths of magnetization vector rotation. The chosen constructive example of this approach is the consideration of the arbitrary oriented plates of cubic m3m ferromagnetic crystals with negative magnetocrystalline anisotropy.

## 2. Directions and paths of magnetization rotation

Let  $\mathbf{m}$  is ort along  $\mathbf{M}$  in the DW volume:  $\mathbf{m} = \mathbf{M}/M$ , where M is a saturation magnetization. Then,  $\mathbf{m}_1$  and  $\mathbf{m}_2$  are orts along  $\mathbf{M}_1$  and  $\mathbf{M}_2$ :  $\mathbf{m}_1 = \mathbf{M}_1/M$ ,  $\mathbf{m}_2 = \mathbf{M}_2/M$ . The angle  $2\alpha$  between them determines DW type [5]  $(2\alpha$ -DW):  $2\alpha = \arccos(\mathbf{m}_1\mathbf{m}_2)$ . The vectors  $\mathbf{m}_1$  and  $\mathbf{m}_2$  coincide with easy magnetization axes (EMA) and determine boundary conditions of DW. In the crystals with negative CMA the EMAs coincide with <111> like crystallographic directions [5]. The all possible combinations of vectors  $\mathbf{m}_1$  and  $\mathbf{m}_2$  ( $C_8^2$ =28 boundary conditions variants) divide into 12, 12 and 4 for the 71°, 109° and 180° DWs respectively.

The orientation of the DW in the unrestricted crystal is defined by the angle  $\lambda$  between direction  $\mathbf{n}_W$  ( $\mathbf{n}_W \perp \Delta \mathbf{m}$  [6]) and plain of vectors  $\Delta \mathbf{m}$  and  $\mathbf{A}$ , where  $\mathbf{A} = \mathbf{m}_\Sigma / (2\cos\alpha)$  and  $\mathbf{A} = [\Delta \mathbf{m} \times \mathbf{P}]/2$  for  $71^0$  or  $109^0$  DW (fig.1a) and  $180^\circ$  DW (fig.1b) respectively, where  $\Delta \mathbf{m} = \mathbf{m}_2 - \mathbf{m}_1$ ,  $\mathbf{m}_\Sigma = \mathbf{m}_1 + \mathbf{m}_2$  and  $\mathbf{P}$  is an ort satisfied the condition  $\mathbf{P}\Delta \mathbf{m} = 0$ . The  $\mathbf{P}$  is useful to orient along any <110> like direction. In this case and only in this case, the vector  $\mathbf{P}$  is collinear with two fold rotation axis and plate of vectors  $\Delta \mathbf{m}$  and  $\mathbf{A}$  coincides with the mirror symmetry plain of the cubic  $\mathbf{m} \mathbf{3} \mathbf{m}$  crystal. The direction of vectors  $\Delta \mathbf{m}$  and  $\mathbf{A}$  determine unambiguously by the orientation of vectors  $\mathbf{m}_1$  and  $\mathbf{m}_2$ . With exception of limit values, all possible orientations of DW plain can be unambiguously described in such ranges of  $\lambda$  as  $-\pi/2 \leq \lambda \leq \pi/2$  [11]. The interrelation between orientation of the  $\mathbf{n}_W$  and the angle  $\lambda$  is given by  $\mathbf{n}_W = \mathbf{A}\cos\lambda + \mathbf{B}\sin\lambda$ , where  $\mathbf{B} = \mathbf{P}$  or  $\mathbf{B} = [\mathbf{m}_1 \times \mathbf{m}_2]/\sin 2\alpha$  for  $180^\circ$  DW or  $71^\circ$  and  $109^\circ$  DW respectively.

In coordinate system connected with DW plain [11]  $O\widetilde{x}\widetilde{y}\widetilde{z}$  ( $[\mathbf{e}_{\widetilde{x}},\mathbf{e}_{\widetilde{y}},\mathbf{e}_{\widetilde{z}}]$ =  $[\Delta \mathbf{m} \times \mathbf{n}_{W}/(2\sin\alpha), \Delta \mathbf{m}/(2\sin\alpha), \mathbf{n}_{W}]$ ) the direction  $\mathbf{m}$  is determined by the polar  $\widetilde{\theta}$  and the azimuth  $\widetilde{\varphi}$  angles, which are counted from the  $O\widetilde{z}$  and  $O\widetilde{x}$  axis respectively (fig.1a-b). Let us set the condition  $\mathbf{m}(\widetilde{z} \to -\infty) \to \mathbf{m}_{1}$  and  $\mathbf{m}(\widetilde{z} \to +\infty) \to \mathbf{m}_{2}$ , that do not restrict model. The spatial distribution of  $\mathbf{m}$  in volume of DW is described by the variable  $\widetilde{\varphi}$  (value  $\widetilde{\theta}$  remained constant in the volume of Bloch DW:  $\widetilde{\theta} = (\mathbf{m} \mathbf{n}_{W}) = \arccos \theta$ , where  $\theta = \cos \lambda \cos \alpha$ ). At  $\theta = 0$  the rotation of  $\mathbf{m}$  realizes in the plain of the DW. Such rotation of the  $\mathbf{m}$  is solely possible for 180° DW. Also, it realizes in the limit cases  $(\lambda = \pm \pi/2)$  for 71° and 109° DW.

There are possible opposite directions of  $\mathbf{m}$  rotation between vectors  $\mathbf{m}_1$  and  $\mathbf{m}_2$  in volume of the DW. The corresponding ranges of  $\widetilde{\varphi}$  change are bounded by the limit values  $\widetilde{\varphi}_1 = \widetilde{\varphi}(\widetilde{z} \to -\infty)$  and  $\widetilde{\varphi}_2 = \widetilde{\varphi}(\widetilde{z} \to +\infty)$ , where  $\widetilde{\varphi}_1 = -\widetilde{\varphi}_0 + (C_R + 1)\pi$ ,  $\widetilde{\varphi}_2 = \widetilde{\varphi}_0$ ,  $\widetilde{\varphi}_0 = \arccos(\cos\alpha\sin\lambda/\sin\widetilde{\theta})$ ,  $\alpha < \widetilde{\varphi}_0 < \pi - \alpha$ . The value  $C_R$  is given by  $C_R = -1$  and 1 for right- (R-) and left-handed (L-) direction of  $\mathbf{m}$  rotation respectively. There are short  $(\Delta \widetilde{\varphi} < \pi)$ , middle  $(\Delta \widetilde{\varphi} = \pi)$  and long  $(\Delta \widetilde{\varphi} > \pi)$  paths (S-, M- and L-path

respectively) of  $\mathbf{m}$  turn in the volume of the DW, where  $\Delta \widetilde{\varphi} = \left| \widetilde{\varphi}_2 - \widetilde{\varphi}_1 \right|$  [11]. For the 71° and 109° DW value of  $\Delta \widetilde{\varphi}$  changes in the range  $2\alpha \leq \Delta \widetilde{\varphi} < \pi$  at  $\pi/2 \geq \widetilde{\lambda} > 0$  and the range  $\pi < \Delta \widetilde{\varphi} \leq 2(\pi - \alpha)$  at  $0 > \widetilde{\lambda} \geq -\pi/2$  for S- and L-path respectively, where  $\widetilde{\lambda} = -C_R \lambda$ . The M-path is the path of  $\mathbf{m}$  turn in the whole range of changes of  $\lambda$   $(-\pi/2 \leq \lambda \leq \pi/2)$  and only in point  $\lambda = 0$  for 180° DW and non-180° DW respectively. The kind of path is determined by the parameter  $C_P = C_R \operatorname{sgn}(\lambda \cos \alpha)$ :  $C_P = -1$ ,  $C_P = 0$  or  $C_P = 1$  for S-, M- or L-path respectively.

Analysis of the above-mentioned paths of m rotation leads to the formulation of the following conclusions about spatial position of this rotation. At the  $0 < \widetilde{\lambda} \le \pi/2$  the **m** in the volume of the 71° DW with S-path paths passes two octants of Cartesian coordinate system Oxyz (based on [100], [010] and [001] directions) which contain  $\mathbf{m}_1$  and  $\mathbf{m}_2$ . In the volume of the 109° DW with S-path the vector  $\mathbf{m}$ passes two or three octants at the  $\tilde{\lambda} = \pi/2$  or  $0 < \tilde{\lambda} < \pi/2$  respectively. At the *M*-path in the volume of the 71° or 109° DW the rotation of **m** passes two or three octants respectively. With exceptions of fixed orientations, the rotation of m in the volume of the 180° DW passes four octants in the whole region  $-\pi/2 \le \lambda \le \pi/2$ . The above-mentioned exceptions are the orientations of 180° DW with  $\lambda = \pi (d/3 - 1/2)$ ,  $d = \overline{0.3}$ , where rotation of **m** passes three octants. In the case of 71° DW with the *L*path, the rotation of **m** in its volume passes two, four, six and four octants at the  $0 \ge \widetilde{\lambda} \ge \mu_1$ ,  $\mu_1 > \widetilde{\lambda} \ge \mu_2$ ,  $\mu_2 > \widetilde{\lambda} > -\pi/2$  and  $\widetilde{\lambda} = -\pi/2$  respectively. The rotation of **m** in the volume of the 109° DW with *L*path passes three, four, five or four octants at the  $0 \ge \widetilde{\lambda} \ge \eta_1$ ,  $\eta_1 > \widetilde{\lambda} \ge \eta_2$ ,  $\eta_2 > \widetilde{\lambda} > -\pi/2$  or  $\widetilde{\lambda} = -\pi/2$ following:  $\mu_1 = \arccos\sqrt{3/[2(5-2\sqrt{3})]} \approx -8^{\circ}48'$ , respectively. These values are the  $\mu_2$ =-arccos $\sqrt{3/[2(5+2\sqrt{3})]} \approx -65^{\circ}6'$ ,  $\eta_1$ =- $\pi/6 = -30^{\circ}$  and  $\eta_2$ =-arctan  $\sqrt{2/3} \approx -39^{\circ}14'$ .

### 3. Energy and structure of the domain walls

minimum of the energy density  $\sigma_S = \sigma S/S_0 = \sigma/\sin\psi$ , where  $\sigma = 2 \Big|_{\widetilde{\varphi}_1}^{\widetilde{\varphi}_2} \{ [A\sin^2\widetilde{\theta}] \cdot [e_A(\widetilde{\theta},\widetilde{\varphi}) - e_A(\widetilde{\theta},\widetilde{\varphi}_1)] \}^{1/2} d\widetilde{\varphi} \Big|$  is the energy density of the DW in the unrestricted crystal [5], A is an exchange constant,  $e_A(\widetilde{\theta},\widetilde{\varphi})$  is the CMA energy density,  $e_A(\widetilde{\theta},\widetilde{\varphi}_1) = K_1/3$  [5,6],  $\psi$  is an angle between plain of the DW and the surface of the (nml) plate,  $S_0$  and S is the area of the DW at the  $(\mathbf{n}_W \mathbf{n}_S) = 0$  and arbitrary orientation of the  $\mathbf{n}_W$  respectively,  $\mathbf{n}_S$  is an ort along the normal of the sample surface:  $\mathbf{n}_S = (\mathbf{e}_1 n + \mathbf{e}_2 m + \mathbf{e}_3 l)/\sqrt{u}$ ,  $u = n^2 + m^2 + l^2$ , n, m and l are the Miller indexes of the sample surface;  $\mathbf{e}_1$ ,  $\mathbf{e}_2$  and  $\mathbf{e}_3$  are the orts along the [100], [010] and [001] directions respectively. In the coordinate system  $O\widetilde{x}\widetilde{y}\widetilde{z}$  the CMA energy density  $e_A = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_1^2\alpha_3^2)$  is given by expressions:

The equilibrium orientation of the DW in the volumetric plate of the magnetically ordered

medium depends on the mutual orientation of the DW plain and the sample surface and corresponds to the

$$e_{A}(\widetilde{\theta},\widetilde{\varphi})/|K_{1}| = \left[\Pi + (1-\Pi)\sin^{2}(\Lambda + \pi/4)\sin^{2}(\Lambda - \pi/4)\right](\Pi - 1) \quad \text{for} \quad 71^{\circ} \text{ DW}, \tag{1a}$$

$$e_{A}(\widetilde{\theta}, \widetilde{\varphi})/|K_{1}| = -(1-\Pi)\cos^{2}\Lambda[1-(1-\Pi)\cos^{2}\Lambda] - [\Pi-(1-\Pi)\sin^{2}\Lambda]^{2}/4 \text{ for } 109^{\circ} \text{ DW},$$
 (1b)

$$e_A(\widetilde{\theta}, \widetilde{\varphi})/|K_1| = \cos^2\widetilde{\varphi}(9 - 7\cos 2\widetilde{\varphi} - 4\sqrt{2}\sin 2\widetilde{\varphi}\sin 3\lambda)/24 - 1/3 \text{ for } 180^\circ \text{ DW},$$
 (1c)

where  $\Pi = \sin^2 \widetilde{\theta} \sin^2 \widetilde{\varphi}$ ,  $\Lambda = \lambda - \lim_{\tau \to 0+} \arctan[\cos \widetilde{\varphi} \tan(\widetilde{\theta} - \tau)]$ ,  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$ —are direction cosines of the **m** in the Oxyz. The following equalities:  $e_A(\widetilde{\theta}, \widetilde{\varphi}) = e_A(\widetilde{\theta}, -\widetilde{\varphi})$  and  $e_A(\widetilde{\theta}, \widetilde{\varphi}) = e_A(\widetilde{\theta}, \pi - \widetilde{\varphi})$  take place at the any value of  $\lambda$  or at the  $\lambda \to -\lambda$  respectively. Based on the last expression, the functions  $\sigma(\lambda)$  of the semi-type DW with opposite directions of the **m** rotation are interrelated by the  $\lambda \to -\lambda$ . For the 180° DW the value of  $e_A(\widetilde{\theta}, \widetilde{\varphi})$  remains constant at the changing of the  $\lambda$  by the angle  $\pi/3$  as well as  $e_A(\widetilde{\theta}, \widetilde{\varphi}) = e_A(\widetilde{\theta}, \pi - \widetilde{\varphi})$  takes place for the any value of  $\lambda$ . Consequently, values of  $\sigma(\lambda)$  for 180° DWs with opposite directions of the **m** rotation are equal.

The function  $S(\lambda)$  has parameters  $\beta = \arccos |\mathbf{n}_S \mathbf{A}|$ ,  $\gamma = \arccos |\mathbf{n}_S \Delta \mathbf{m}/(2\sin \alpha)|$  and  $\nu = \operatorname{sgn}[(\mathbf{A}\mathbf{n}_S)(\mathbf{B}\mathbf{n}_S)]$ :

$$S(\lambda) = S_0 / \sqrt{1 - \sin^2 \gamma \cos^2 \left[ \left( \nu + |\nu| - 1 \right) \lambda - b \right]}, \tag{2}$$

where  $b = \lim_{\tau \to 0+} \arctan \left[ \sqrt{(-\cos 2\beta - \cos 2\gamma)/2} / \cos(\beta - \tau) \right]$ . The parameters  $\beta$  and  $\gamma$  (each of them is determined in the region  $[0, \pi/2]$ ) are interrelated:  $\cos 2\beta + \cos 2\gamma + \cos 2\phi = -1$ , where  $\phi = \arccos \beta$ . The condition for  $\phi$  to have real value restricts a totality of possible parameters:  $-2 \le \cos 2\beta + \cos 2\gamma \le 0$  (fig.2). Internal part of this region is determined by the  $0 < b < \pi/2$ . The value  $\nu = 0$  is met at the  $\beta = \pi/2$  and/or  $\phi = \pi/2$ , i.e. at the bound of the region where parameters  $\beta$  and  $\gamma$  are defined. Geometrical sense of the  $\beta$  and  $\gamma$  is that they are angles of reflections of vectors  $\mathbf{A}$  and  $\Delta \mathbf{m}$  accordingly from the sample plate normal. The angle  $\phi$  is an interfacial angle between the sample plain and plain of the vectors  $\mathbf{A}$  and  $\Delta \mathbf{m}$  (fig.1c). As it was mentioned in the initial definitions our model is invariant when vectors  $\mathbf{m}_1$  and  $\mathbf{m}_2$  are transposed. Here, all values and functions are identical with exceptions of value  $\nu$  sign inversion. For the 180° DW the values of the  $\beta$  and  $\phi$ , and else  $\nu$  depend on choice of the vector  $\mathbf{P}$ . Ambiguous manner of it choice is determined by the six possible <110> like directions and corresponding six functions  $\sigma_s = \sigma_s(\lambda)$ . These dependence are differed by the shift  $k\pi/3$ , where k=1, 2, 3 taking into account looping at the boundaries of argument region. Unambiguous description of the 180° DW requires the following definition of the  $\mathbf{P}$  choice:

$$\mathbf{P} = \begin{cases}
\mathbf{P}_{1} = s_{1}(-s_{3}\mathbf{e}_{2} + s_{2}\mathbf{e}_{3}) / \sqrt{2} & \text{at } |m| < |n| < |l| \text{ or } |m| > |n| > |l| \text{ or } |l| = |m| \neq |n| \\
\mathbf{P}_{2} = s_{2}(-s_{1}\mathbf{e}_{3} + s_{3}\mathbf{e}_{1}) / \sqrt{2} & \text{at } |l| < |m| \text{ or } |l| > |m| > |n| \text{ or } |n| = |l| \neq |m| \\
\mathbf{P}_{3} = s_{3}(-s_{2}\mathbf{e}_{1} + s_{1}\mathbf{e}_{2}) / \sqrt{2} & \text{at } |n| < |l| < |m| \text{ or } |n| > |l| > |m| \text{ or } |m| = |n| \neq |l| \\
\mathbf{P}_{k} (k = 1, 2 \text{ or } 3) & \text{at } |n| = |m| = |l| \text{ and } \gamma = 0 \\
\mathbf{P}_{k} \perp \mathbf{n}_{S} & \text{at } |n| = |m| = |l| \text{ and } \gamma \neq 0
\end{cases} \tag{3}$$

where  $s_k = \sqrt{3} \Delta \mathbf{m} \mathbf{e}_k / 2$  (k = 1, 2, 3).

The area remains constant  $S(\lambda) = S_0$  at the  $\gamma = 0$ . In the other cases (at the  $\gamma > 0$ ), dependences  $S(\lambda)$  has minimum at the value of  $\lambda$ , that is equal to the  $\lambda_0 = (\nu + |\nu| - 1)(b - \pi/2)$  or  $\lambda_0 = \pm \pi/2$  accordingly at the b > 0 or b = 0 and maximum value (including limit case  $S(\lambda) \to \infty$  at the  $\gamma = \pi/2$ ) when  $\lambda$  equals  $\lambda_m = (\nu + |\nu| - 1)b$  or  $\lambda_m = \pm \pi/2$  at the  $b < \pi/2$  or  $b = \pi/2$  respectively. There are the symmetry  $S(\lambda_0 + \Delta \lambda) = S(\lambda_0 - \Delta \lambda)$  and  $S(\lambda_m + \Delta \lambda) = S(\lambda_m - \Delta \lambda)$ . Dependences  $S(\lambda)$  for DWs with different  $\nu$  signs are interrelated by the  $\lambda \to -\lambda$ . For symmetrical reflections of this DW from the  $\lambda = 0$  the lower areas S correspond to the ranges containing the  $\lambda_0$  (i.e.  $-\pi/2 < \lambda \nu < 0$ ):  $S(\lambda) < S(-\lambda)$ . For the  $\nu = 0$  only the values  $\lambda_0$  and  $\lambda_m$  equals to the 0 or  $\pm \pi/2$ .

In general case, dependence  $\sigma_S(\lambda)$  in the unique manner is defined by the  $\beta$ ,  $\gamma$ , and  $\nu$  or  $\tilde{\nu}$  for the non-180° or 180° DW respectively, where  $\tilde{\nu}=\nu C_R$ . The rotation direction does not correspond to the specific value of the energy: both R- and L- rotations are energetically more preferable for different orders of domains (interrelated by the  $\mathbf{m}_1\leftrightarrow\mathbf{m}_2$ ) if all others things are equal. The equilibrium DWs with opposite directions of the  $\mathbf{m}$  rotations have the identical energy density  $\sigma_S$  when the function  $S(\lambda)$  is even for 71° or 109° DW ( $\nu=0$ ) and always for the 180° DW ( $\beta=0$ ). Equilibrium 71° or 109° DW have the rotation of  $\mathbf{m}$  in their plain ( $\beta=0$ ) only at the b=0 ( $\phi=\beta+\gamma=\pi/2$ ). The general criterion of the coincidence of the equilibrium values  $\sigma_S$  of DWs with opposite directions of the  $\mathbf{m}$  rotations is given by  $\nu\beta=0$ . Here, orientations of the equilibrium 71° or 109° DWs with opposite directions of the  $\mathbf{m}$  rotation is given by the angles  $\lambda_I=-\lambda_II$ .

The dependence  $\sigma_S(\lambda)$  can coincide with the one  $\sigma(\lambda)$  (in the unrestricted crystal) completely at the  $\gamma=0$  (curve 1 on the fig.3a-b) or in the points  $\lambda=\lambda_0$  (curve 2 on the fig.3a-b). If the value  $\lambda_0$  corresponds to the equilibrium value in the unrestricted crystal then equilibrium DW will be the same too. This case realizes when  $\lambda_0=39^\circ44'\cdot C_R$  or  $\lambda_0=\pm\pi/2$ , and else when  $\lambda_0=-10^\circ9'\cdot C_R$  or  $\lambda_0=-C_R\pi/2$  for 71°, and else for 109° DW respectively.

If  $v\mathcal{G} \neq 0$  takes place then equilibrium  $71^0$  and  $109^0$  DWs with opposite directions of the **m** rotations (at  $0 < b < \pi/2$ ) have different equilibrium values  $\sigma_s$  (3-4 on the fig.3a-b). If equilibrium DW with specific direction of the rotation presents in the region  $-\pi/2 < \lambda v < 0$  then it is more preferable then this DW with opposite direction.

The spatial distribution  $\widetilde{z}$  of the  $\widetilde{\varphi}$  of the  $\mathbf{m}$  rotation in the volume of the DW is given by the  $\widetilde{z} = \int_{\varpi}^{\widetilde{\varphi}} \left\{ \left[ A \sin^2 \widetilde{\theta} \right] / \left[ e_A \left( \widetilde{\theta}, \widetilde{\zeta} \right) - e_A \left( \widetilde{\theta}, \widetilde{\varphi}_1 \right) \right] \right\}^{1/2} d\widetilde{\zeta} \text{ , where } \varpi = 0 \text{ and } \pi \text{ for the } R\text{- and } L\text{- rotation in the volume}$ 

of the DW respectively. The peculiarities of the spatial distribution of the  $\mathbf{m}$  is given by the number of the reflection points of the distribution  $\widetilde{z}(\widetilde{\varphi})$ . This number is given by the number of the zero values of the function  $\partial e_A(\widetilde{\theta},\widetilde{\varphi})/\partial \widetilde{\varphi}$  in the ranges  $\widetilde{\varphi}_1 \leq \widetilde{\varphi} \leq \widetilde{\varphi}_2$  and  $\widetilde{\varphi}_2 \leq \widetilde{\varphi} \leq \widetilde{\varphi}_1$  for R- and L-rotations respectively. The  $\mathbf{m}$  distribution in the volume of the 180° DW is always non-monotone (has more than one reflection point) and the number of reflection points  $\widetilde{\varphi}_k^*$  of the  $\widetilde{z}(\widetilde{\varphi})$  equals three in the range  $[-\pi/2 \leq \lambda \leq \pi/2]$ :

$$\widetilde{\varphi}_{1}^{*} = \varpi + \Xi \sec^{-1} \left[ \sqrt{I \left( I + \sqrt{3} \right)} \rho^{2} + 3\rho \left( 9 - \cos 6\lambda \right) + I \left( I - \sqrt{3} \right) \left( 4 + 8\sin^{2} 3\lambda + 9\sin^{4} 3\lambda \right) / \left( 4\sqrt{\rho} \right) \right]$$

$$\tag{4a}$$

$$\widetilde{\varphi}_{2}^{*} = \varpi - \Xi \sec^{-1} \left[ \sqrt{\left(-1 - I\sqrt{3}\right)} \rho^{2} + 3\rho \left(9 - \cos 6\lambda\right) + I\left(I + \sqrt{3}\right) \left(4 + 8\sin^{2} 3\lambda + 9\sin^{4} 3\lambda\right) / \left(4\sqrt{\rho}\right) \right]$$

$$\tag{4b}$$

$$\widetilde{\varphi}_{3}^{*} = \varpi + \Xi \sec^{-1} \left| \sqrt{4 + 12\rho + \rho^{2} + (8 + 3\rho)\sin^{2}3\lambda + 9\sin^{4}3\lambda} / (2\sqrt{2\rho}) \right|$$
(4c)

where  $\rho = \left[ \left( 26 + 36 \sin^2 3\lambda + 27 \sin^4 3\lambda \right) \sin^2 3\lambda - 8 + 10 \sqrt{-\sin^2 3\lambda} \left( 8 + 11 \sin^2 3\lambda + 8 \sin^4 3\lambda \right) \right]^{1/3}$ ,  $I = \sqrt{-1}$  and  $\Xi = \operatorname{sgn}(\sin 3\lambda)$ . Here and hereinafter the reflection points  $\widetilde{\varphi}_2^*$  and  $\widetilde{\varphi}_3^*$  are the most remote [5] reflection points of the function  $\widetilde{z}(\widetilde{\varphi})$ . The expressions (4) give the real values of the  $\widetilde{\varphi}_k^*$ , where  $k = \overline{1,3}$ . Here and hereinafter all functions of complex argument are token in sense of value principal.

The **m** distribution in the volume of the 71° and 109° DW has the central reflection point  $\widetilde{\varphi}_1^* = \varpi$ . The roots of the equality  $\partial e_A(\widetilde{\theta},\widetilde{\varphi})/\partial \widetilde{\varphi} = 0$  in the considering region of  $\widetilde{\varphi}$  gives the following reflection points. They are given for 71° DW:

$$\widetilde{\varphi}_{3}^{*} = -\widetilde{\varphi}_{2}^{*} + 2\varpi = \arccos\left\{-f_{1}(\widetilde{\lambda}, \widetilde{\theta})/\left[36(7 - \cos 4\widetilde{\lambda})\sin \widetilde{\theta}\right]\right\} + \varpi$$
 (5a),

$$\widetilde{\varphi}_{5}^{*} = -\widetilde{\varphi}_{4}^{*} + 2\varpi = \arccos\left\{-g_{1}\left(\widetilde{\lambda}, \widetilde{\theta}\right) / \left[36\left(7 - \cos 4\widetilde{\lambda}\right)\sin \widetilde{\theta}\right]\right\} + \varpi$$
 (5b)

and for the 109° DW:

$$\widetilde{\varphi}_{3}^{*} = -\widetilde{\varphi}_{2}^{*} + 2\varpi = \min \left\{ \arccos \left[ f_{2}(\widetilde{\lambda}, \widetilde{\theta}) \right], \arccos \left[ g_{2}(\widetilde{\lambda}, \widetilde{\theta}) \right] \right\} + \varpi$$
(6a),

$$\widetilde{\varphi}_{5}^{*} = -\widetilde{\varphi}_{4}^{*} + 2\varpi = \arccos \left\{ f_{3}(\widetilde{\lambda}, \widetilde{\theta}) \right\} + \varpi \tag{6b}.$$

The analytical expressions for the functions  $f_1(\widetilde{\lambda},\widetilde{\theta})$ ,  $f_2(\widetilde{\lambda},\widetilde{\theta})$ ,  $g_1(\widetilde{\lambda},\widetilde{\theta})$  and  $g_2(\widetilde{\lambda},\widetilde{\theta})$  are presented in the Appendix A. The range of the orientations of the DW where these reflection points exist is restricted by the condition of real value:  $\operatorname{Im} \widetilde{\varphi}_k^* = 0$ , where  $k = \overline{1,5}$ . For the 71° DW the number of the reflection points equals one and five in the regions  $\pi/2 \geq \widetilde{\lambda} \geq \widetilde{\lambda}_1$  at  $\operatorname{Im} f_1(\widetilde{\lambda},\widetilde{\theta}) \neq 0$  and  $\widetilde{\lambda}_1 > \widetilde{\lambda} > -\pi/2$  at  $\operatorname{Im} f_1(\widetilde{\lambda},\widetilde{\theta}) = 0$  respectively. For the 109° DW the number of the reflection points equals one, three and five in the regions  $\pi/2 \geq \widetilde{\lambda} \geq \lambda_2$ ,  $\lambda_2 > \widetilde{\lambda} \geq \lambda_3$  and  $\lambda_3 > \widetilde{\lambda} > -\pi/2$  respectively. The values of the angles  $\widetilde{\lambda}_2$  and  $\widetilde{\lambda}_3$  are given by the equations  $f_2(\widetilde{\lambda},\widetilde{\theta}) = 1$  and  $f_3(\widetilde{\lambda},\widetilde{\theta}) = 1$ . Analytical expression for the function  $f_3(\widetilde{\lambda},\widetilde{\theta})$  has been presented in the Appendix A. The values of these angles are the following:  $\widetilde{\lambda}_1 \approx -29.366^\circ$ ,  $\widetilde{\lambda}_2 \approx 27.784^\circ$  and  $\widetilde{\lambda}_3 \approx -66.669^\circ$ .

The monotone **m** distribution is symmetrical, i.e.  $\widetilde{\varphi}(\widetilde{z}) - \varpi = -\widetilde{\varphi}(-\widetilde{z}) + \varpi$ . The **m** distribution is symmetrical in the volumes of the 71° and 109° DW at their arbitrary orientation and else in the volume of the 180° DW at the orientation along the vector  $\mathbf{n}_W$  along the directions like <112>, with extra condition  $\mathbf{n}_W \perp \Delta \mathbf{m}$ :  $\pm (s_1 s_3 \mathbf{e}_1 + s_2 s_3 \mathbf{e}_2 - 2\mathbf{e}_3)/\sqrt{6}$ ,  $\pm (s_1 s_2 \mathbf{e}_1 - 2\mathbf{e}_2 + s_3 s_2 \mathbf{e}_3)/\sqrt{6}$  and  $\pm (-2\mathbf{e}_1 + s_2 s_1 \mathbf{e}_2 + s_3 s_1 \mathbf{e}_3)/\sqrt{6}$ . They correspond to the orientations of the 180° DW with  $\lambda = 0$  or  $\pm \pi/3$ .

#### 4. Limit and unrestricted width of the domain walls

It was determined that some Bloch DW orientations correspond to the "infinity" width of the DW [3]. Appearing of the unstable DWs can be the consequence of the applying of the external magnetic field

leading to the reorientation of the DW plane and/or reorientation of the domain magnetization directions. So, the general analysis of the DW width is required. This can be obtained analytically based on the expressions (4-6).

The analytical expressions for the width of the DW were obtained for the monotone [5-6] and non-monotone (180° DW only) [3] **m** distribution in the volume of the DW. For the 71° and 109° DWs with monotone **m** distributions the expressions are given by:  $\bar{\delta}=2\sqrt{6}L/\sqrt{5-3\cos4(\tilde{\theta}-|\lambda|)}$  and  $\bar{\delta}=4\sqrt{3}L/[1+3\cos2(\tilde{\theta}-|\lambda|)]$ , where  $L=\Delta\tilde{\varphi}\sin\tilde{\theta}$  is the length of the path of the **m** rotation [11]. Expressions for the 180° DW with non-monotone **m** distribution was presented in [3]. Calculation of the width of DWs with remained types and non-monotone **m** distribution requires determination of the reflection points  $\tilde{\varphi}_2^*$  and  $\tilde{\varphi}_3^*$ . These points were determined by the graphical or numerical methods [3]. Taking into account (5) and (6) we obtained that DW width is given by the expression for R –rotation:

$$\delta = \delta_0 \left\{ \frac{\left(\widetilde{\varphi}_2 - \widetilde{\varphi}_3^*\right)}{\sqrt{\overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_3^*)} \cdot \overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_1)} + \frac{\left(\widetilde{\varphi}_2^* - \widetilde{\varphi}_1\right)}{\sqrt{\overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_2^*)} \cdot \overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_1)} \right\} \sin \widetilde{\theta} + \widetilde{z} \left(\widetilde{\varphi}_3^*\right) \cdot \widetilde{z} \left(\widetilde{\varphi}_2^*\right)$$
(7a)

and for the L – rotation:

$$\delta = \delta_0 \left\{ \frac{\left(\widetilde{\varphi}_1 - \widetilde{\varphi}_3^*\right)}{\sqrt{\overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_3^*)} + \overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_1)} + \frac{\left(\widetilde{\varphi}_2^* - \widetilde{\varphi}_2\right)}{\sqrt{\overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_2^*)} + \overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}_1)} \right\} \sin \widetilde{\theta} + \widetilde{z} \left(\widetilde{\varphi}_3^*\right) - \widetilde{z} \left(\widetilde{\varphi}_2^*\right), \tag{7b},$$

where  $\delta_0 = \sqrt{A/K_1}$  and  $\overline{e}_A(\widetilde{\theta}, \widetilde{\varphi}) = e_A(\widetilde{\theta}, \widetilde{\varphi})/|K_1|$ .

In the case of the non-monotone  $\mathbf{m}$  distribution the function  $\widetilde{\varphi}(\widetilde{z})$  can contain reflection points with  $\partial^2 e_A(\widetilde{\theta},\widetilde{\zeta})/\partial\widetilde{\zeta}^2>0$ . These points lead to the growth (unrestricted in the limit case) of the DW width. In the scope of the current model the unrestricted DW width realizes when  $\mathbf{m}$  passes EMAs. This situation takes place at the critical orientations  $\widetilde{\lambda}=\widetilde{\lambda}_J^C$  of the DW plain:  $\mathbf{n}_W\left(-C_R\widetilde{\lambda}_J^C\right)\cdot\mathbf{m}_1=\mathbf{n}_W\left(-C_R\widetilde{\lambda}_J^C\right)\cdot\mathbf{u}_J$ , where  $\mathbf{u}_J$  is the ort along EMA ( $\mathbf{u}_J\neq\mathbf{m}_1$ ;  $\mathbf{u}_J\neq\mathbf{m}_2$ ), where J- is an integer. There are the following critical orientations:  $\widetilde{\lambda}_1^C=-\pi/4$  and  $\widetilde{\lambda}_2^C=-\pi/2$  for the 71° DW;

 $\widetilde{\lambda}_1^C = 0$ ,  $\widetilde{\lambda}_2^C = -\arccos(1/\sqrt{3})$  and  $\widetilde{\lambda}_3^C = -\pi/2$  for the 109° DW;  $\widetilde{\lambda}_1^C = -\pi/2$ ,  $\widetilde{\lambda}_2^C = -\pi/6$ ,  $\widetilde{\lambda}_3^C = \pi/6$  and  $\widetilde{\lambda}_4^C = \pi/2$  for the 180° DW. The DWs with such plain orientations are unstable, i.e. they disintegrate when obtain such orientation.

In the such cases the 71° DW divides into three 71° DWs at the  $\widetilde{\lambda} = \widetilde{\lambda}_1^C$  or two 109° DWs and one 71° DW at the  $\widetilde{\lambda} = \widetilde{\lambda}_2^C$ . The 109° DW divides into the two 71° DWs at the  $\widetilde{\lambda} = \widetilde{\lambda}_1^C$ , two 109° DWs at the  $\widetilde{\lambda} = \widetilde{\lambda}_2^C$  or two 71° DWs and one 109° DW at the  $\widetilde{\lambda} = \widetilde{\lambda}_3^C$ . The 180° DW divides into the 71° DW and 109° DW at any critical orientation. The conservation of the length [5,11] of the  $\mathbf{m}$  rotation takes place:  $L = \sum_k L_k$  as well as the conservation of the energy density  $\sigma_S = \sum_k \sigma_S^k$  and  $\Delta \widetilde{\varphi} = \sum_k \Delta \widetilde{\varphi}_k$ , where  $L_k = \Delta \widetilde{\varphi}_k \sin \widetilde{\theta}_k$ . Here,  $\Delta \widetilde{\varphi}_k$  and  $\sigma_S^k$  are the corresponding parameters of DWs which born after disintegration of the initial DW (these DWs are numbered by the k). All paths of  $\mathbf{m}$  rotation of the initial DWs are L- or M-paths. All paths of the new DWs are the S-paths.

The stability and restrictions of the unstable DWs is leading by the taking into account of the extra components of energy [3]. For example, it can be magnetostriction energy term. Thus, consideration of the DWs in the crystal with small magnetostriction is of interest [21,22]. Also, taking into account demagnetization energy term leads to stabilization of the above-mentioned DWs as well as rising of the new types of unstable DW types [23].

#### 5. Schema of the domain walls degeneration in the (nml) plates

The above mentioned theory allows to describe and to analyze the energy and the structure (plain orientation, magnetization distribution, DW width) of the 71°, 109° and 180° DW with arbitrary boundary conditions in the plate with arbitrary oriented surface. The analysis of these expressions leads to formulate the main laws for these DWs. Equilibrium DWs in the restricted crystal corresponds to the minimum of the energy density  $\sigma_s$  and do not coincide with such DWs in the unrestricted crystal in general case. It leads to dividing of the semi-type equilibrium DWs into the  $Q_{2\alpha}$  groups. The semi-types

DWs from different groups have different areas and values  $\sigma_s$ . Each group includes q semi-types DWs with the identical values  $\sigma_s$  but different  $\mathbf{A}$  and/or  $\Delta \mathbf{m}$ . The criterion of the equal values  $\sigma_s$  of semi-type DWs with specific  $\mathbf{m}$  rotation path spatial position is the equality of their areas. The last is determined by the identical angle pairs  $\beta$  and  $\gamma$ . In general case the values q are different for different groups. In the arbitrary (nml) plate two  $71^{\circ}$  DWs (or two  $109^{\circ}$  DWs) belong to group if their vectors  $\mathbf{A}$  (or  $\Delta \mathbf{m}$ ) are collinear to the directions  $(\mathbf{e}_i + \mathbf{e}_j)/\sqrt{2}$  and  $(\mathbf{e}_i - \mathbf{e}_j)/\sqrt{2}$  (in the (nml) plates with  $S_iS_j=0$ , where  $S_i=(\mathbf{n}_S\mathbf{e}_i)$ ,  $S_j=(\mathbf{n}_S\mathbf{e}_j)$ ,  $i\neq j$ , i,j=1,2,3), directions  $(\mathbf{e}_i\pm\mathbf{e}_j)/\sqrt{2}$  and  $(\mathbf{e}_i\pm\mathbf{e}_k)/\sqrt{2}$  (in the (nml) plates with  $S_i=\pm S_j$  or  $S_i=\mp S_j=\mp S_k$  or simultaneous  $S_i=0$  and  $\pm S_j=\mp S_k$ , where  $S_k=(\mathbf{n}_S\mathbf{e}_k)$ ,  $k\neq i,j$ , k=1,2,3). Two  $180^{\circ}$  DW belong to the group if their vectors  $\Delta \mathbf{m}$  are collinear to the directions  $(\mathbf{e}_i\pm\mathbf{e}_j+\mathbf{e}_k)/\sqrt{3}$  and  $(-\mathbf{e}_i\pm\mathbf{e}_j+\mathbf{e}_k)/\sqrt{3}$  (in the (nml) plates with  $S_i=0$  or  $S_j=-S_k$ ), directions  $(\mathbf{e}_i+\mathbf{e}_j-\mathbf{e}_k)/\sqrt{3}$  and  $(-\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_k)/\sqrt{3}$  (in the (nml) plates with  $S_i=0$  or  $S_j=S_k$ ) or directions  $(\mathbf{e}_i-\mathbf{e}_j-\mathbf{e}_k)/\sqrt{3}$  and  $(-(\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_k))/\sqrt{3}$  (in the (nml) plates with  $S_i=0$  or  $S_j=S_k$ ) or directions  $(\mathbf{e}_i-\mathbf{e}_j-\mathbf{e}_k)/\sqrt{3}$  and  $(-(\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_k))/\sqrt{3}$  (in the (nml) plates with  $S_i=0$  or  $S_j=S_k$ ) or directions  $(\mathbf{e}_i-\mathbf{e}_j-\mathbf{e}_k)/\sqrt{3}$  and  $(-(\mathbf{e}_i+\mathbf{e}_j+\mathbf{e}_k))/\sqrt{3}$  (in the (nml) plates with  $S_i=0$  or  $S_j=S_k$ ).

All possible orientations of plate are described by the six combination of Miller indexes: (100), (110), (111), (nnl), (nm0) and (nml), where n, m and l are the nonzero values with different magnitudes. Results of application of the above-mentioned criteria to the all possible DWs are summarized in table 1-3. The number of groups  $Q_{2\alpha}$  in the (nml) plates growths with reflection of the sample surface normal  $\mathbf{n}_s$  from the high-symmetric crystallographic directions <001>, <110> and <111>. The minimal number of this groups realizes in the (001) plate.

In the spatially restricted crystal 71° DW corresponds to specific 109° DW:  $\beta' = \gamma''$  and  $\gamma' = \beta''$ , where angles with one and two accents relates to the 71° or 109° DW respectively. Vector **B** of the 71° DW always coincides with the vector **A** of some another 71° DW in the (*nml*) plate: angle  $\phi$  of the first DW coincides with the angle  $\beta$  of the second one. These 71° DW have collinear vectors  $\Delta \mathbf{m}$  and

identical angles  $\gamma$ . There are pairs of semi-types non-180° DWs with the same  $\phi$  (and different  $\beta$  and/or  $\gamma$ ) in the (nml) plates which do not contain <110> like directions and contain <112> like directions. They are (nm0) and (nml) plates with  $D = (|n| - |m|)(|m| - |l|)(|l| - |n|) \neq 0$  and  $T = (2|m| - |l \pm n|)(2|n| - |m \pm l|)(2|l| - |n \pm m|) = 0$ .

The DWs with opposite **m** rotation directions (at the same domain order) but the same energy realize in the samples with D=0: (100), (110), (111) and (nnl) plates. For the 109° DWs this totality of plates is wider. It contains also DWs with <100> like directions in their plane (nml=0: (100) plates). When vector **A** of the one 71° DW coincides with the vector **B** of another 71° DW then these DWs can have equal energies at the opposite **m** rotations only simultaneously. The equilibrium 71° and 109° DWs have **m** rotation in their plane in samples with D=0. The equilibrium 180° DWs with different orientations of their planes realize in the samples with <110> or <112> like directions in their plain (TD=0,  $T \neq D$ ) if they have <111> like direction reflected by the angle 40° from the sample normal.

The equilibrium DWs identical to ones in the unrestricted crystal realize in the samples with D=0 (they are DWs with **m** rotation in their planes) and also in the (nml) plates satisfied the conditions:

$$(10.83 \, ||n| - |m|) (10.83 \, ||m| - |l|) (10.83 \, ||l| - |n|) (10.83 \, ||m| - |n|) (10.83 \, ||n| - |m|) (10.83 \, ||n| - |l|) = 0$$
 (8a)

or

$$(1.436n - |m|)(1.436m - |l|)(1.436l - |n|)(1.436m - |n|)(1.436l - |m|)(1.436n - |l|) = 0$$
(8b)

for the 71° and 109° DW respectively. In the specially oriented (001), (110), (111), (112) and the (210) plates they are only 180° DWs ( $\phi$ =90°;  $\beta$ =90°;  $\gamma$ =0°) in the (111) plate, 109° DW ( $\phi$ =90°;  $\beta$ =90°;  $\gamma$ =0°) in the (110) plate and the 71° DW ( $\phi$ =90°;  $\beta$ =90°;  $\gamma$ =0°) in the (001) plate (curve 1 on the fig.3).

As a rule, there are in the restricted bulk crystals DW orientation changes as well as the energy density growths ( $\sigma_s > \sigma$ ) in relation with unrestricted crystals (curves 3-4 on the fig.3). Here, partial or full removing of the semi-type DWs degeneration takes place in the (nml) plates.

The equilibrium orientations of the DWs in the above-mentioned plates have been investigated by the numerical integration. Generally, the equilibrium DWs with the *S*-path of **m** rotation have lower values of the  $\sigma_s$  as well as the values of width  $\delta$ . There are several exceptions of this rule. The 71° DW ( $\phi$ =45°;  $\beta$ =45°;  $\gamma$ =90°) and the 109° DW ( $\phi$ =71.565°;  $\beta$ =26.565°;  $\gamma$ =71.565°) with the *R* – rotation and *L* - path are have the lower energies. They realize in (001) and (210) plate respectively. One more exception is the 109° DW ( $\phi$ =60°;  $\beta$ =45°;  $\gamma$ =60°) with *R* – rotation in the (110) plate. The unique peculiarity of this DW is very small value  $|\lambda|$ <3.7° at the equilibrium orientation.

The DWs with some received equilibrium orientations were experimentally observed in (110)[9,10], (111) [15,16] and (112) plates of iron yttrium garnet [9,16], nickel [10,13] and  $U_3P_4$  single crystal [15]. Their parameters are italicized in the appropriate tables of the Appendix B. The unstable DWs are bold in the Appendix B.

## 6. Conclusions

The typical for the unrestricted crystal degeneration of the semi-type DWs is removed in spatially restricted bulk crystal. The bulk (*nml*) plate contains restrict number of the DWs with different values of the area and the energy. This number is determined by the sample orientation and depends on the magnetization vector orientations of the domains relatively to sample surface. It growth with reflections of sample normal from the high-symmetrical <001>, <110> and <111> like orientations The minimal number of DWs with different energies is realized in the (001) plate.

The most DWs identical to the ones in the unrestricted crystal are possible in the bulk (nml) plates with orthogonal planes of domains magnetization directions and the sample surface. This DWs have absolutely minimal values of the areas and energies among the all semi-types DWs.

All DWs with opposite directions of the magnetization rotation are equal energy at the rotation in the DW plane. There are equal energy 71° DWs with opposite directions of the magnetization rotation in cases when <110> directions are parallel with the sample plane. There are also equal energy 109° DWs

with opposite directions of the magnetization rotation in cases when <110> or <100> directions are parallel with the sample plane. All 180° DWs with opposite directions of rotation are equal energy in the arbitrary oriented (*nml*) plate.

The spatially restricted sample allows equilibrium 71° and 109° DWs with the long path of magnetization rotation. All typical specially oriented plates allow appearing of the unstable DWs which can be considered as the way of the magnetization reversal of the sample.

**Table 1.** The degeneracy q of the plaine Bloch 180° DWs in the (nml) plates

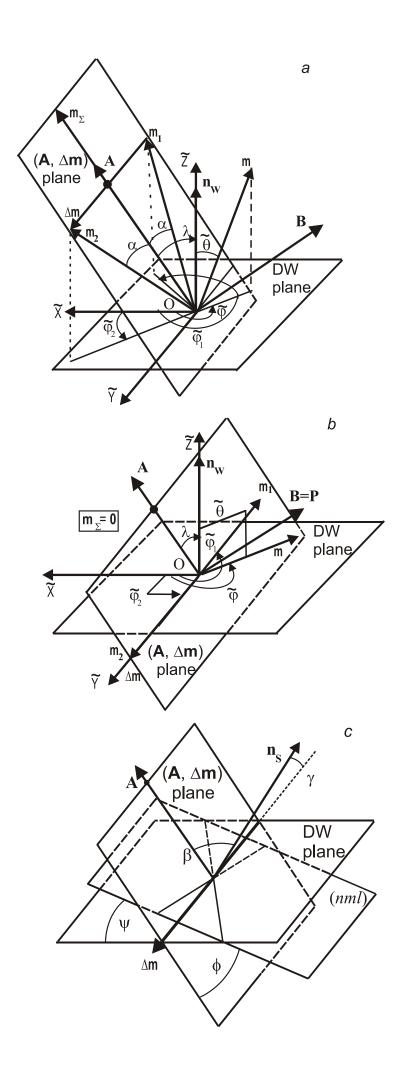
							<i>n</i> ≥0, <i>m</i> ≥0	), <i>l</i> ≥	20					
	n=m=	0;	n = m		m = l	<b>≠</b>	<i>m≠l</i> ; <i>m</i> , <i>l</i> ≠0	•	$n=m\neq l\neq 0$	);	$n\neq m\neq l$ ; $n\neq l$	;		
$\pm \Delta \mathbf{m}$	<i>l≠</i> 0		=i		n = 0	)	n = 0		$n \neq 0$		$n, m, l \neq 0$			
	γ,°	q	γ,°	q	γ,°	q	γ, rad	q	γ, rad	q	γ, rad	q		
[111]			0	1	35.26	2	$\arccos\left[\frac{ m+l }{\sqrt{3u}}\right]$	2	$\arccos\left[\frac{ l+2n }{\sqrt{3u}}\right]$	1	$\arccos\left[\frac{ n+m+l }{\sqrt{3u}}\right]$	1		
[1 11]	54.74	4					[ √3u ]		$\arccos\left[\frac{ l }{\sqrt{3u}}\right]$	2	$\arccos\left[\frac{ m+l-n }{\sqrt{3u}}\right]$	1		
[1 1 1]			71	3	90	2	$\arccos\left[\frac{ m-l }{\sqrt{3u}}\right]$				$\arccos\left[\frac{ n-m+l }{\sqrt{3u}}\right]$	1		
[11 ]									[√3 <i>u</i> ]		$\arccos\left[\frac{ l-2n }{\sqrt{3u}}\right]$	1	$\arccos\left[\frac{ n+m-l }{\sqrt{3u}}\right]$	1
$Q_{180}$	1		2		2		2		3		4			

**Table 2.** The degeneracy q of the plaine Bloch 71° and 109° DWs in the (nml) plates containing the <110> like directions in the plain (nml).

(, 4			>0 >0 >0	`		
$\pm \mathbf{m}_{\Sigma}(\pm \Delta \mathbf{m})$			n≥0, m≥0, l≥0	)		
for $2\alpha = 71^{\circ}$	$n=m=0; l\neq 0$		$n = m \neq l = 0$		<i>n≠m; n,m≠0; l=</i> 0	
or						
$\pm \Delta \mathbf{m} (\pm \mathbf{m}_{\Sigma})$	$\beta\{\gamma\}$ ,° for $2\alpha=71$ °		$\beta(\gamma)$ ,° for $2\alpha=71$ °		$\beta\{\gamma\}$ , rad for $2\alpha=71^{\circ}$	
for $2\alpha = 109^{\circ}$	or $\gamma\{\beta\}$ ,° for	q	or $\gamma\{\beta\}$ ,° for	q	or $\gamma\{\beta\}$ , rad for	q
	2α=109°		2α=109°		2α=109°	
[110]([001])			0{90}	2	$\left[ \left  n+m \right  \right] \left\{ \left  \pi/2 \right  \right\}$	2
	00(0)	4			$\arccos\left[\frac{ n+m }{\sqrt{2u}}\right] \{\pi/2\}$	
[1 1 0]([001])	90{0}		90{90}	2	$\arccos\left[\frac{ n-m }{\sqrt{2u}}\right]\left\{\pi/2\right\}$	2
					[ √2 <i>u</i> ]	
[011]([100])					$\arccos[m]/\sqrt{2u}$	4
[01 1]([100])	45{90}	8	60{45}	8	$\{\arccos[n]/\sqrt{u}]\}$	
[101]([010])	43 (30)	O	007437	0	$arccos[n]/\sqrt{2u}$	4
$[\overline{1} 01]([010])$					$\{\arccos[m /\sqrt{u}]\}$	
$Q_{71}$ or $Q_{109}$	2		3		4	

**Table 3.** The degeneracy q of the plaine Bloch 71° and 109° DWs in the (nml) plates not containing the <110> like directions in the plain (nml).

$\pm \mathbf{m}_{\Sigma} (\pm \Delta \mathbf{m})$			<i>n</i> ≥0, <i>m</i> ≥0, <i>l</i> ≥0		
for $2\alpha = 71^{\circ}$	n=m=l		$n = m \neq l \neq 0; n \neq 0$	0	$n \neq m \neq l$ ; $n \neq l$ ;
or					$n,m,l \neq 0$
$\pm \Delta \mathbf{m} (\pm \mathbf{m}_{\Sigma})$	$\beta$ { $\gamma$ },° for 2 $\alpha$ =71°		$\beta$ { $\gamma$ },rad for 2 $\alpha$ =71°		$\beta\{\gamma\}$ , rad for $2\alpha=71^{\circ}$
for $2\alpha=109^{\circ}$	or $\gamma\{\beta\}$ ,° for	q	or $\gamma\{\beta\}$ , rad for	q	or $\gamma\{\beta\}$ , rad for $q$
	2α=109°		2α=109°		2α=109°
[110]([001])			$\arccos[n]\sqrt{2}/\sqrt{u}$	2	$\left  \operatorname{arccos}[n+m]/\sqrt{2u} \right  = 2$
			$\{\arccos[l]/\sqrt{u}\}$	2	$\{\arccos[l]/\sqrt{u}\}$
[101]([010])	35.26 {54.74}	6	[		$\left  \arccos  n+l /\sqrt{2u} \right  = 2$
	(34.74)	U	$arccos n + l / \sqrt{2u}$	4	$\{\arccos[m]/\sqrt{u}\}$
[011]([100])			$\{\arccos[n]/\sqrt{u}\}$		$arccos m+l/\sqrt{2u}$ 2
					$\{\operatorname{arccos}[n]/\sqrt{u}\}$
[1 10]([001])			$\pi/2$	2	$\arccos[n-m]/\sqrt{2u}$ 2
			$\{\arccos[l]/\sqrt{u}\}$	2	$\{\arccos[l]/\sqrt{u}\}$
[0 1 1]([100])	90 {54.74}	6	[		$arccos m-l/\sqrt{2u}$ 2
	(37.77)	U	$\arcsin  n-l /\sqrt{2u}$	4	$\{\arccos[n]/\sqrt{u}\}$
$[10\overline{1}]([010])$			$\{\arccos[n]/\sqrt{u}\}$		$\arccos\left n-l\right /\sqrt{2u}$ 2
					$\{ \operatorname{arccos}[m] / \sqrt{u} ] \}$
$Q_{71}$ or $Q_{109}$	2		4		6



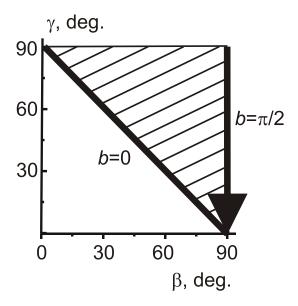
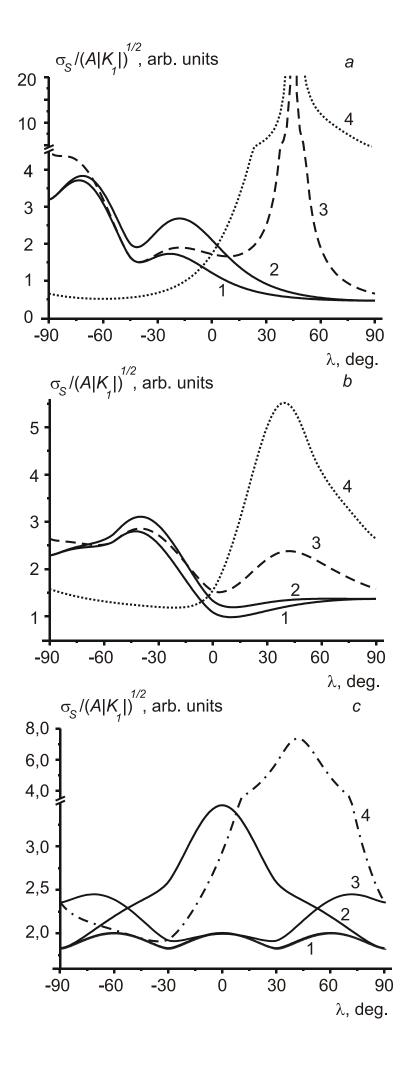


Fig.1. The coordinate system for the  $180^{0}$  DW (a) and non- $180^{0}$  DW (b); the definition of the angles describing the mutual position of the  $\mathbf{m}_{1}$ ,  $\mathbf{m}_{2}$ , DW plane and sample (nml) plane.

Fig.2. The area of possible angles  $\beta$  and  $\gamma$ . The bold line mark special values were  $\nu = 0$ .



- Fig.3. The examples of the orientation dependences of the 71° DW (a), 109° DW (b) and the 180° DW (c):
- a). The orientation dependences of the energy density of the 71 degree DW with right-handed (1-3) and the left-handed (4) magnetization rotation. The parameters  $(\nu, \beta, \gamma)$  are:  $\nu = 0$  (curves 1-2) and  $\nu = 1$  (curves 3-4);  $\beta = 90$  deg. and  $\gamma = 0$  deg. (curve 1),  $\beta \approx 35.264$  deg. and  $\gamma \approx 54.735$  deg. (curve 2), and else  $\beta = 45$  deg. and  $\gamma = 90$  deg. (curves 3-4).
- **b).** The orientation dependences of the energy density of the 109 degree DW with right-handed (1-3) and the left-handed (4) magnetization rotation. The parameters  $(\nu, \beta, \gamma)$  are:  $\nu = 0$  (curves 1-2) and  $\nu = 1$  (curves 3-4);  $\beta = 90$  deg. and  $\gamma = 0$  deg. (curve 1),  $\beta \approx 54.735$  deg. and  $\gamma \approx 35.264$  deg. (curve 2), and else  $\beta = 45$  deg. and  $\gamma = 60$  deg. (curve 3-4).
- c). The orientation dependences of the energy density of the 109 degree DW with arbitrary direction of the magnetization rotation. The parameters  $(\nu, \beta, \gamma)$  are:  $\nu = 0$  (curves 1-3) and  $\nu = 1$  (curve 4);  $\beta = 90$  deg. and  $\gamma = 0$  deg. (curve 1),  $\beta \approx 35.264$  deg. and  $\gamma \approx 54.736$  deg. (curve 2),  $\beta = 90$  deg. and  $\gamma \approx 39.232$  deg. (curve 3), and else  $\beta \approx 43.089$  deg. and  $\gamma \approx 75.037$  deg. (curve 4).

# Appendix A.

$$\begin{split} f_1\left(\widetilde{\lambda},\widetilde{\theta}\right) &= -162^{1/3}\left(1+I\sqrt{3}\right)t_1 + 2\left(18\sqrt{3}\right)^{2/3}\left(1-I\sqrt{3}\right)\left[11\cos 4\widetilde{\lambda} + \cos 2\widetilde{\theta}\left(23+7\cos 4\widetilde{\lambda}\right)-5\right]/t_1 + \\ &+ 36\cos\widetilde{\theta}\sin 4\widetilde{\lambda}\;, \\ f_2\left(\widetilde{\lambda},\widetilde{\theta}\right) &= \csc\widetilde{\theta}\left[36\cos\widetilde{\theta}\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda} + 6\cos 4\widetilde{\lambda} + 1\right)\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda} + 6\cos 4\widetilde{\lambda}\right) + 1\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda} + 6\cos 4\widetilde{\lambda}\right) + 1\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right] + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right) + \\ &+ 3\cos^2\theta\left(3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda}\right) + \left[6^{5/3}\left(I\sqrt{3} + 1\right)\left(-166 + 23\cos 2\widetilde{\lambda}\right) + 6\cos 4\widetilde{\lambda}\right) + \\ &+ 3\cos^2\theta\left(3\cos 2\widetilde{\lambda} + \cos^2\theta\right) + \cos^2\theta\left(3\cos 2\widetilde{\lambda} + \cos^2\theta\right) + \cos^2$$

$$+4\cos^2\widetilde{\theta}\left(2\cos^2\widetilde{\lambda}+2\cos^2\widetilde{\lambda}-17\right)+9\cos^2\widetilde{\lambda}\right)/\sqrt[3]{t_2}+\left(I\sqrt{3}-1\right)\sqrt{6^4t_2}/\left[72\left(3\cos^22\widetilde{\lambda}-5-14\cos^2\widetilde{\lambda}\right)\right],$$

$$f_3(\widetilde{\lambda},\widetilde{\theta}) = \csc\widetilde{\theta} \left\{ 18\cos\widetilde{\theta} \left( 3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda} \right) - \left[ 6^{5/3} \left( 23\cos 2\widetilde{\lambda} + 6\cos 4\widetilde{\lambda} + 6\cos 4\widetilde{\lambda}$$

$$+4\cos 2\widetilde{\theta} \left(28\cos 2\widetilde{\lambda}+21\cos 4\widetilde{\lambda}-17\right)+9\cos 6\widetilde{\lambda}-166\right)\right]/\sqrt[3]{t_{2}}+\sqrt[3]{6^{4}t_{2}}/\left[36\left(3\cos^{2} 2\widetilde{\lambda}-5-14\cos 2\widetilde{\lambda}\right)\right],$$

$$g_1(\widetilde{\lambda},\widetilde{\theta}) = -162^{1/3} \left(1 - I\sqrt{3}\right) t_1 + 2\left(18\sqrt{3}\right)^{2/3} \left(1 + I\sqrt{3}\right) \left[11\cos 4\widetilde{\lambda} + \cos 2\widetilde{\theta}\left(23 + 7\cos 4\widetilde{\lambda}\right) - 5\right] / t_1 + 36\cos\widetilde{\theta}\sin 4\widetilde{\lambda},$$

$$\begin{split} g_2\left(\widetilde{\lambda},\widetilde{\theta}\right) &= \csc\widetilde{\theta} \left\{ 36\cos\widetilde{\theta} \left( 3\sin 4\widetilde{\lambda} - 14\sin 2\widetilde{\lambda} \right) + \left[ 6^{5/3} \left( 1 - I\sqrt{3} \right) \left( -166 + 23\cos 2\widetilde{\lambda} + 6\cos 4\widetilde{\lambda} \right) + 4\cos 2\widetilde{\theta} \left( 28\cos 2\widetilde{\lambda} + 2\log 4\widetilde{\lambda} - 17 \right) + 9\cos 6\widetilde{\lambda} \right) \right] / \sqrt[3]{t_2} - \sqrt[3]{6^4t_2} \left( 1 + I\sqrt{3} \right) / \left[ 72 \left( 3\cos^2 2\widetilde{\lambda} - 5 - 14\cos 2\widetilde{\lambda} \right) \right] , \end{split}$$

where 
$$t_1 = \sqrt{6} \left[ 2(\cos 4\widetilde{\lambda} - 7)^2 (250 + 189 \cos 2\widetilde{\theta} + 25 \cos 4\widetilde{\theta} + 407 \cos 6\widetilde{\theta} + (782 + 82 \cos 2\widetilde{\theta} + 642 \cos 4\widetilde{\theta} + 434 \cos 6\widetilde{\theta}) \cos 4\widetilde{\lambda} - 216 \cos^2 \widetilde{\theta} \cos 2\widetilde{\lambda} \right]^{1/2} - 72 \left( 1 + 8 \cos 2\widetilde{\lambda} + \cos 4\widetilde{\lambda} \right) \sin 4\widetilde{\lambda} \cos \widetilde{\theta}^{1/3},$$

$$\begin{split} t_2 = & \left[ -6 \left( 7 + 28 \cos 2\widetilde{\lambda} - 3 \cos 4\widetilde{\lambda} \right)^2 \left[ 6394 + 804 \cos 2\widetilde{\theta} - 4080 \cos 4\widetilde{\theta} - 2416 \cos 6\widetilde{\theta} - 2\cos 2\widetilde{\lambda} \left( 4213 + 8112 \cos 2\widetilde{\theta} + 1140 \cos 4\widetilde{\theta} + 224 \cos 6\widetilde{\theta} \right) + 24\cos 4\widetilde{\lambda} \left( 181 + 228 \cos 2\widetilde{\theta} + 330 \cos 4\widetilde{\theta} + 98\cos 6\widetilde{\theta} \right) + 27 \left( 97 + 96 \cos 2\widetilde{\theta} + 56 \cos 4\widetilde{\theta} \right) \cos 6\widetilde{\lambda} - 54 \left( 6\cos 2\widetilde{\theta} - 5 \right) \cos 8\widetilde{\lambda} - 81 \cos 10\widetilde{\lambda} \right] \right]^{1/2} + 36 \left[ -4\cos 3\widetilde{\theta} \times \left( 35\sin 2\widetilde{\lambda} + 4\sin 4\widetilde{\lambda} + 7\sin 6\widetilde{\lambda} \right) - \cos \widetilde{\theta} \left( 302\sin 2\widetilde{\lambda} - 50\sin 4\widetilde{\lambda} + 6\sin 6\widetilde{\lambda} + 9\sin 8\widetilde{\lambda} \right) \right]. \end{split}$$

# Appendix B. Energy density and structure of the plane Bloch DW (equilibrium states in regular and unstable in bold) in the (nml)-plates of crystal with "negative" CMA

Table B.1. Energy density and structure of the plane Bloch DW in (001)-plate

Int.angl.		DW typ	ie	M turr	ı type	]	DW energy	density	and structur	re
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$ar{\delta}$ ,	$\overline{\sigma}/\sin\psi$ ,
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.
					S-	8.923	155.257	36.078	3.206	1.659
	71	45.000	90.000	R-	L-	-40.035	264.585	85.035	16.655	1.503
				L-	S-	-60.232	78.333	74.769	4.085	0.510
45.000					S-	±9.216	167.077	83.497	9.944	0.988
				R/L-		∓56.247	240.904	53.991	14.512	3.156
	109	90.000	45.000		L-	∓90.000	250.529	45.000	2∞	3.239
					S-	±90.000	70.529	90.000	4.264	0.461
				R/L-		∓90.000	289.471	90.000	2∞	3.197
	71	90.000	0.000		L-	∓39.734	264.227	90.000	16.341	1.497
90.000					S-	±90.000	109.471	90.000	3.309	1.368
	109	0.000	90.000	R/L-	L-	∓90.000	250.529	90.000	2∞	2.290
						-90.000	180.000	90.000	∞	1.829
	180	35.264	54.736	R/L-	М-	90.000	180.000	90.000	∞	1.829

Here and hereinafter the next abridgements are assumed: Int.angl. – interfacial angle.

Table B.2. Energy density and structure of the plane Bloch DW in (110)-plate

Int.angl.		DW typ	oe .	M turn	type	I	OW energy	density	and structur	re
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$\overline{\delta}$ ,	$\overline{\sigma}/\sin\psi$ ,
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.
					S-	± 35.217	101.603	54.783	3.650	0.738
	71	90.000	90.000	R/L-	L-	<del>+</del> 35.616	258.947	54.385	13.028	1.899
0.000	109	90.000	90.000	R/L-	S-	± 8.481	168.093	81.519	10.143	0.994
						7.212	180.000	82.788	8.135	2.004
	180	90.000	90.000	R/L-	<i>M</i> -	-7.212	180.000	82.788	8.135	2.004
						-39.890	264.414	86.389	16.502	1.500
				R-	L-	-89.046	289.464	60.554	24.056	3.683
	71	60.000	45.000		S-	-67.577	74.828	74.248	4.163	0.497
60.000				L-	L-	39.380	263.801	45.275	15.987	2.107
					S-	3.649	174.847	42.482	12.083	1.509
	109	45.000	60.000	R-	L-	-63.489	244.648	82.427	12.026	2.508
				L-	S-	-20.809	151.798	61.095	7.614	1.183
					S -	±90.000	70.529	90.000	4.264	0.461
	71	0.000	90.000	R/L-		<b>∓90.000</b>	289.471	90.000	2∞	3.197
					L-	<del>+</del> 42.813	267.728	42.813	20.792	2.263
					S-	±10.146	165.799	90.000	9.710	0.981
90.000	109	90.000	0.000	R/L-	L-	<b>∓90.000</b>	250.529	90.000	2∞	2.290
						90.000	180.000	90.000	œ	1.829
						-90.000	180.000	90.000	∞	1.829
	180	54.736	35.264		<i>M</i> -	33.555	180.000	61.240	12.301	2.103
				R/L-		-33.555	180.000	61.240	12.301	2.103

Table B.3. Energy density and structure of the plane Bloch DW in (111)-plates

Int.angl.		DW typ	oe .	M turr	ı type	DW energy density and structure					
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$\overline{\delta}$ ,	$\overline{\sigma}/\sin\psi$ ,	
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.	
					S-	±45.000	90.000	54.736	3.848	0.667	
	71	90.000	54.736	R/L-		∓37.881	261.939	59.911	14.663	1.743	
35.264					L-	<b>790.000</b>	289.471	35.264	2∞	5.538	
					S-	3.236	175.428	51.500	12.357	1.309	
	109	54.736	90.000	R-	L-	-55.894	240.697	69.371	15.067	2.734	
				L-	S-	-16.188	157.696	70.924	8.453	1.060	
					S-	±90.000	70.529	90.000	4.264	0.461	
	71	35.264	54.736	R/L-		<b>∓90.000</b>	289.471	90.000	2∞	3.197	
					L-	∓41.383	266.147	52.221	18.299	1.907	
						±11.389	164.102	55.530	9.421	1.192	
	109	54.736	35.264	R/L-	S-	±90.000	109.471	90.000	3.309	1.368	
90.000					L-	<b>∓90.000</b>	250.529	90.000	2∞	2.290	
70.000						30.000	180.000	90.000	00	1.829	
						-30.000	180.000	90.000	00	1.829	
	180	90.000	0.000	R/L-	М-	90.000	180.000	90.000	00	1.829	
						-90.000	180.000	90.000	00	1.829	
						-90.000	180.000	90.000	00	1.829	
	180	19.471	70.529	R/L-	М-	90.000	180.000	90.000	00	1.829	

Table B.4. Energy density and structure of the plane Bloch DW in (112)-plate

Int.angl.		DW typ	e	M turr	ı type	]	DW energy	density	and structur	re
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$\overline{\mathcal{\delta}}$ ,	$\bar{\sigma}/\sin\psi$ ,
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.
					S-	31.406	107.224	45.760	3.568	0.885
	71	73.221	65.905	R-	L-	-38.543	262.773	71.709	15.218	1.581
30.000				L-	S-	-47.953	87.195	63.271	3.901	0.595
					S-	5.180	172.695	61.003	11.285	1.146
	109	65.905	73.221	R-	L-	-54.729	239.996	61.881	27.308	2.926
				L-	S-	-12.994	161.932	78.284	9.077	1.008
					S-	±75.348	72.324	56.043	4.221	0.564
	71	90.000	35.264	R/L-		∓39.020	263.363	68.685	15.646	1.608
					L-	∓90.000	289.471	54.736	2∞	3.916
54.736					М-	0.000	180.000	35.264	2∞	1.886
	109	35.264	90.000	R-	L-	-62.402	244.147	82.334	12.067	2.514
				L-	S-	-30.159	140.885	65.423	5.882	1.230
	180	48.190	61.875	R/L-	М-	-33.877	180.000	76.604	12.090	1.897
						-41.193	265.931	62.512	18.038	1.697
				R-	L-	-89.304	289.467	73.851	25.611	3.335
	71	30.000	65.905		S-	-78.089	71.709	84.047	4.236	0.468
73.221				L-	L-	42.657	267.559	33.643	20.456	2.768
					S-	8.733	167.745	63.427	10.074	1.099
	109	65.905	30.000	R-	L-	-89.351	250.525	73.499	28.622	2.391
				L-	S-	-12.062	163.190	70.189	9.273	1.046

Table B.4 (continue).

Int.angl.		DW typ	oe .	M turr	n type	DW energy density and structure					
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$ar{\delta}$ ,	$\overline{\sigma}/\sin\psi$ ,	
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.	
					S-	±90.000	70.529	90.000	4.264	0.461	
	71	54.736	35.264	R/L-		∓90.000	289.471	90.000	2∞	3.197	
					L-	∓40.417	265.036	63.924	17.079	1.668	
					S-	±90.000	109.471	90.000	3.309	1.368	
	109	35.264	54.736	R/L-	L-	<b>∓90.000</b>	250.529	90.000	2∞	2.290	
90.000						90.000	180.000	90.000	$\infty$	1.829	
						-90.000	180.000	90.000	$\infty$	1.829	
	180	70.529	19.471	R/L-	М-	30.469	180.000	73.304	17.257	1.910	
						-30.469	180.000	73.304	17.257	1.910	
						90.000	180.000	90.000	$\infty$	1.829	
	180	0.000	90.000	R/L-	М-	-90.000	180.000	90.000	∞	1.829	

Table B.5. Energy density and structure of the plane Bloch DW in (210)-plate

Int.angl.		DW typ	oe .	M turr	ı type	DW energy density and structure					
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$\overline{\mathcal{\delta}}$ ,	$\overline{\sigma}/\sin\psi$ ,	
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.	
					S-	25.437	117.449	46.129	3.441	0.964	
18.435	71	71.565	90.000	R-	L-	-38.250	262.406	70.185	14.967	1.599	
				L-	S-	-45.000	90.000	63.435	3.848	0.609	
	109	90.000	71.565	R/L-	S-	±8.616	167.906	81.829	10.106	0.993	
						-87.347	289.413	52.951	18.988	4.078	
				R-	L-	-39.978	264.518	85.509	16.595	1.501	
	71	50.769	63.435		S-	-62.439	77.153	74.452	4.111	0.506	
				L-	L-	37.616	261.601	27.500	14.452	3.272	
					S-	6.138	171.353	59.185	10.896	1.158	
	109	63.435	50.769	R-	L-	-58.607	242.230	72.126	12.801	2.650	
50.769				L-	S-	-13.481	161.279	73.294	8.977	1.032	
						-27.172	180.000	73.213	12.859	1.921	
						27.172	180.000	73.213	12.859	1.921	
	180	90.000	39.232	R/L-	М-	-90.000	180.000	50.769	8	2.362	
						90.000	180.000	50.769	8	2.362	
	180	43.089	75.037	R/L-	М-	-35.034	180.000	76.414	11.455	1.907	

Table B.5 (continue)

Int.angl.		DW typ	oe .	M turi	ı type	DW energy density and structure					
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$ar{\delta}$ ,	$\overline{\sigma}/\sin\psi$ ,	
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.	
						-89.100	289.465	72.465	24.346	3.363	
				R-	L-	-41.519	266.301	59.954	18.494	1.744	
	71	18.435	90.000		S-	-77.351	71.861	84.214	4.232	0.469	
				L-	L-	44.803	269.802	26.368	32.394	3.651	
					S-	±9.940	166.082	86.871	9.761	0.983	
	109	90.000	18.435	R/L-	L-	∓90.000	250.529	71.565	2∞	2.414	
71.565						-89.771	289.471	71.642	31.056	3.370	
				R-	L-	-39.798	264.304	87.676	16.406	1.498	
	71	71.565	26.565		S-	-77.876	71.752	75.951	4.235	0.480	
				L-	L-	39.651	264.128	63.560	16.256	1.672	
					М-	0.000	180.000	26.564	2∞	2.434	
	109	26.565	71.565	R-	L-	-87.404	250.473	74.014	21.857	2.402	
				L-	S-	-59.607	117.239	79.645	3.687	1.333	

Table B.6. Energy density and structure of the plane Bloch DW in (123)-plate

Int.angl.		DW typ	e	M turr	type		DW energ	y density	y density and structure			
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$ar{\delta}$ ,	$\overline{\sigma}/\sin\psi$ ,		
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.		
					S-	31.251	107.467	49.325	3.565	0.838		
	71	79.107	74.499	R-	L-	-37.757	261.781	64.585	14.563	1.671		
19.107				L-	S-	-42.495	92.617	60.068	3.800	0.643		
				R-	S-	6.264	171.176	68.360	10.849	1.069		
	109	74.499	79.107	L-	S-	-11.156	164.419	85.447	9.473	0.985		
					S-	36.902	99.327	40.875	3.686	0.903		
				R-		-39.144	263.514	79.393	15.762	1.524		
	71	67.792	57.689		L-	-86.963	289.395	42.706	18.316	4.820		
					S-	-54.360	82.051	66.790	4.005	0.553		
				L-	L-	36.189	259.725	41.290	13.409	2.323		
					S-	4.275	173.966	53.887	11.722	1.252		
40.893	109	57.689	67.792	R-	L-	-56.661	241.144	70.260	14.028	2.705		
				L-	S-	-15.193	159.003	71.474	8.641	1.051		
	180	77.396	51.887	R/L-	М-	-28.291	180.000	80.438	14.090	1.859		
	180	49.107	90.000	R/L-	М-	-32.189	180.000	81.296	13.484	1.857		
						-39.273	263.671	77.726	15.885	1.532		
				R-	L-	-89.635	289.470	55.547	28.784	3.880		
55.462	71	79.107	36.699		S-	-64.873	75.981	64.339	4.137	0.535		
33.402				L-	L-	38.899	263.213	59.796	15.535	1.735		
	109	36.699	79.107		S-	0.444	179.373	36.279	17.020	1.821		
			,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	R-	L-	-62.534	244.209	82.343	12.059	2.513		
				L-	S-	-28.697	142.492	64.464	6.140	1.225		

Table B.6 (continue).

Int.angl.	DW type			M turr	type	DW energy density and structure					
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$ar{\delta}$ ,	$\overline{\sigma}/\sin\psi$ ,	
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.	
						-40.671	265.331	70.912	17.377	1.587	
				R-	L-	-89.077	289.464	68.547	24.222	3.445	
	71	40.893	57.689		S-	-73.933	72.695	81.142	4.213	0.475	
				L-	L-	41.389	266.154	35.212	18.307	2.615	
					S-	7.338	169.678	54.660	10.483	1.211	
				R-		-88.261	250.504	68.803	23.805	2.467	
	109	57.689	40.893		L-	-66.105	245.765	82.585	12.108	2.496	
				L-	S-	-14.127	160.417	64.779	8.848	1.095	
67.792						-29.405	180.000	79.306	16.671	1.862	
						29.405	180.000	79.306	16.671	1.862	
	180	90.000	22.208	R/L-	М-	-90.000	180.000	67.792	8	1.976	
						90.000	180.000	67.792	∞	1.976	
	180	29.206	72.025	R/L-	М-	-82.740	180.000	74.655	10.573	1.941	
						-40.325	264.928	71.945	16.975	1.576	
				R-	L-	-89.774	289.471	79.237	31.121	3.255	
	71	55.462	36.699		S-	-81.899	71.071	83.846	4.251	0.466	
79.107				L-	L-	40.336	264.940	56.325	16.987	1.800	
					S-	8.113	168.603	34.872	10.248	1.722	
	109	36.699	55.462	R-	L-	-89.184	250.523	79.773	27.500	2.330	
				L-	S-	-65.800	114.359	81.008	3.535	1.350	

Table B.6 (continue).

Int.angl.	DW type			M turn type		DW energy density and structure				
$\phi$ ,	$2\alpha$ ,	$\beta$ ,	γ,	Rota-	Path	λ,	$\Delta\widetilde{arphi}$ ,	ψ,	$\overline{\mathcal{\delta}}$ ,	$\overline{\sigma}/\sin\psi$ ,
deg.	deg.	deg.	deg.	tion		deg.	deg.	deg.	arb.units.	arb.units.
						-41.770	266.583	54.630	18.869	1.856
				R-	L-	-89.582	289.470	79.509	28.113	3.254
	71	19.107	74.499		S-	-82.381	71.009	86.444	4.253	0.464
79.107				L-	L-	43.240	268.185	35.133	21.847	2.696
	109	74.499	19.107		S-	9.603	166.545	72.840	9.845	1.027
				R-	L-	-89.812	250.528	79.158	34.695	2.332
				L-	S-	-10.863	164.818	76.888	9.540	1.008

### Reference

- [1] R. Vakhitov, A Yumaguzin, J. Magn. Magn. Mater. 215-216, 52 (2000)
- [2] R.M. Sabitov, R.M. Vakhitov, Sov. Phys. Izv. Vuzov. Fizika, 8 (1988) 51
- [3] B.A. Lilley. Phil.Mag. 41 (1950) 792.
- [4] L.D. Landau, E.M. Lifshitz. Sow. Phys. 8 (1935) 153.
- [5] A. Hubert. Theory of Domain Walls. Springer, New York, 1974.
- [6] Hubert A., Shafer R. Magnetic domains. The analysis of magnetic microstructures. Berlin, Springer-Verlag, 1998.
- [7] A. Aharoni, J.P. Jakubovics. J.Magn.Magn.Mater. **104-107** (1992) 353
- [8] F.B. Humphrey, M. Redjdal. J.Magn.Magn.Mater. 133 (1994) 11
- [9] V.L. Vlasko-Vlasov, L.M. Dedukh, V.I. Nikitenko. JETP 71 (1976) 2291
- [10] J. Peters, W. Treimer. J.Magn.Magn.Mater. 241 (2002) 240
- [11]. O. Antonyuk, V.F Kovalenko, O.V. Tychko. Journ. Alloys and Compaund. 369 (2004) 112
- [12] V.A. Gurevich et al., sov. phys.: sol. stat. phys. 19 (1977) 761
- [13] O. Scharpf, R. Seifert, H. Strothmann. J.Magn.Magn.Mater. 13 (1979) 239
- [14] Stupakiewicz A., Maziewski A., Davidenko I., Zablotskii V. Phys.Rev.B. 64 (2001) 064405
- [15] A. Szewczyk, Z. Henkie. J.Magn.Magn.Mater. **81** (1989) 277
- [16] L.A. Pamyatnykh, G.S. Kandaurova, M.A. Shamsutdinov, V.V. Plavski, B.N. Filippov. J.Magn.Magn.Mater. 234 (2001) 469
- [17] L.M. Dedukh, V.I. Nikitenko, V.T. Synogach. JETP Letters. 45 (1987) 491
- [18] V. V. Randoshkin, A. M. Saletskii, N. N. Usmanov, D. B. Chopornyak. Physics of the Solid State 44 (2002) 899
- [19] V. Baryakhtar, V. Lvov, D. Yablonsky, JETP 87 (1984) 1863
- [20] B. M. Tanygin, O. V. Tychko, Physica B: Condensed Matter, 404, 21, 4018-4022 (2009).
- [21] J.F. Dillon, J.P. Remeika, E.M. Gyorgy. Appl. Phys. Lett. 9 (1966) 147
- [22] S.T. Chui and V.N. Ryzhov J. Magn. Magn. Mater. 182 (1998) 25

[23] S.A. D'yachenko et al., Physics of Solid State, 2008, 50, 1, 32-42.